Week 3 Test Exercise

(a)

AIC: log(s2) + 2k/n

Small model: AIC0 = log(s02) + 2p0/n

Big model: AIC1 = log(s12) + 2p1/n

AIC: Choose small model if

AIC0 < AIC1

log(s02) + 2p0/n < log(s12) + 2p1/n

log(s02/ s12) < 2/n\*(p1 – p0)

**s02/ s12 < e2/n\*(p1 – p0)**

(b)

Given:

ex ≈ 1 + x *(if x is small)*

If n is very large, (2/n)( p1 – p0) is small. Therefore

e2/n\*(p1 – p0) ≈ 1 + 2/n\*(p1 – p0) *(if n is very large)*

Substituting this expression into the right side of the result from part (a) yields

s02/ s12 < 1 + 2/n\*(p1 – p0) *(for n very large)*

s02/ s12 - 1 < 2/n\*(p1 – p0)

**(s02 - s12)/ s12 < 2/n\*(p1 – p0)** *(for n very large)*

(c)

n

s2 = 1/(n-1) **Σ** ei2 = 1/(n-1)\*e’e

i=1

=> s02 = 1/(n-1)\*eR’eR, s12 = 1/(n-1)\*eU’eU

Substituting theses expressions into the result from part (b) yields

(1/(n-1)\*eR’eR - 1/(n-1)\*eU’eU)/ (1/(n-1)\*eU’eU) < 2/n\*(p1 – p0)

**(eR’eR - eU’eU)/ eU’eU < 2/n\*(p1 – p0)**

(d)

F-test (formula from Lecture 2.4.2 slides):

F = (eR’eR - eU’eU)/g

eU’eU/(n – k)

where k is the number of explanatory factors in the unrestricted model, and g is the number of explanatory factors removed from the unrestricted model to create the restricted model.

Under this test, we believe there is significant evidence to suggest that β ≠ 0 (so the unrestricted model is preferred) if F > Fcritical. Therefore a larger model is preferred if F > Fcritical, and we stay with (prefer) a smaller model if F < Fcritical.

Let Fcritical = 2. Then a smaller model is preferred if F < 2:

(eR’eR - eU’eU)/g < 2

eU’eU/(n – k)

In this case, with p1 factors in the unrestricted model and p0 in the restricted model, we get

(eR’eR - eU’eU)/(p1 – p0) < 2

eU’eU/(n – p1)

(eR’eR - eU’eU)/( eU’eU) < 2(p1 – p0)/(n – p1)

If n is very large, n – p1 ≈ n. Then we get the desired result:

**(eR’eR - eU’eU)/( eU’eU) < 2(p1 – p0)/n**

is our condition for preferring a restricted model when doing an F-test with Fcritical = 2 (and when n is very large), just as it was our condition for preferring a restricted model when using the AIC (when n is very large).